# Sketching Linear Classifiers Over Data Streams

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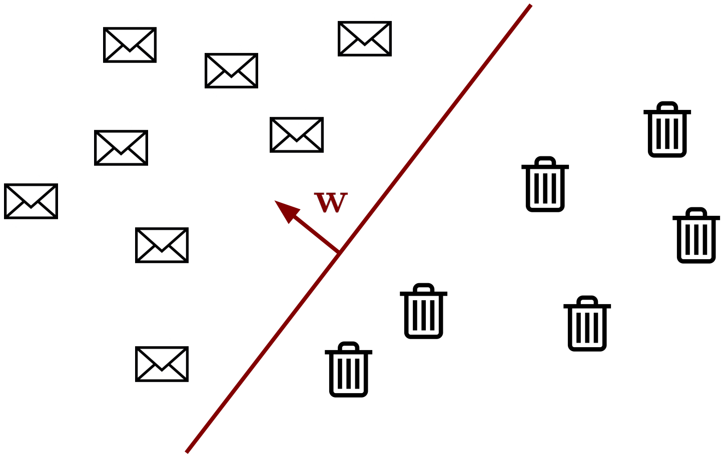
# High-dimensional linear classifiers on streams

**Ubiquitous:** spam detection, ad click prediction, network traffic classification, ...

**Fast:** computationally cheap inference and updates

**Adaptive:** updated online in response to changing data distributions




**Problem: high memory usage**

Lots of features $\Rightarrow$ more expressive classifiers,
BUT more memory needed to store weights

# Example: Traffic classification with limited memory

Network packet

```
Version:    IPv4
Src:   136.0.1.1
Dest:  129.0.1.1
...
```

→

Features

```
 Src[:1] = 136
Dest[:1] = 129
 Src[:2] = 136.0
Dest[:2] = 129.0
...
```

→ **Classifier**

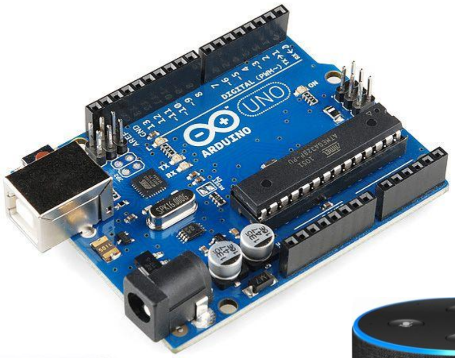
→ Accept

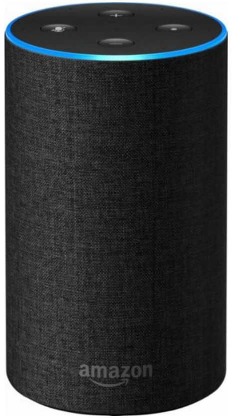
→ Reject (filter)

- Want classifiers that adhere to strict memory budgets (e.g., 1MB)
- But also want accuracy: more features, feature combinations

network switch

# More broadly: Online learning on memory-constrained devices



# Problem: How to restrict memory usage while preserving accuracy?


"I don't think you're getting the point of this exercise."

Proposal 1: Use only most informative features?

- In streaming setting, often don't know feature importance *a priori*
- Feature importance can change over time (e.g., spam classification)

Proposal 2: Use only most frequent features?

- Most frequent ≠ most informative

# Sketches for memory-limited stream processing

Long line of work on memory-efficient *sketches* for stream processing

e.g., identifying the *k* most frequent items in a stream ("heavy hitters" problem)

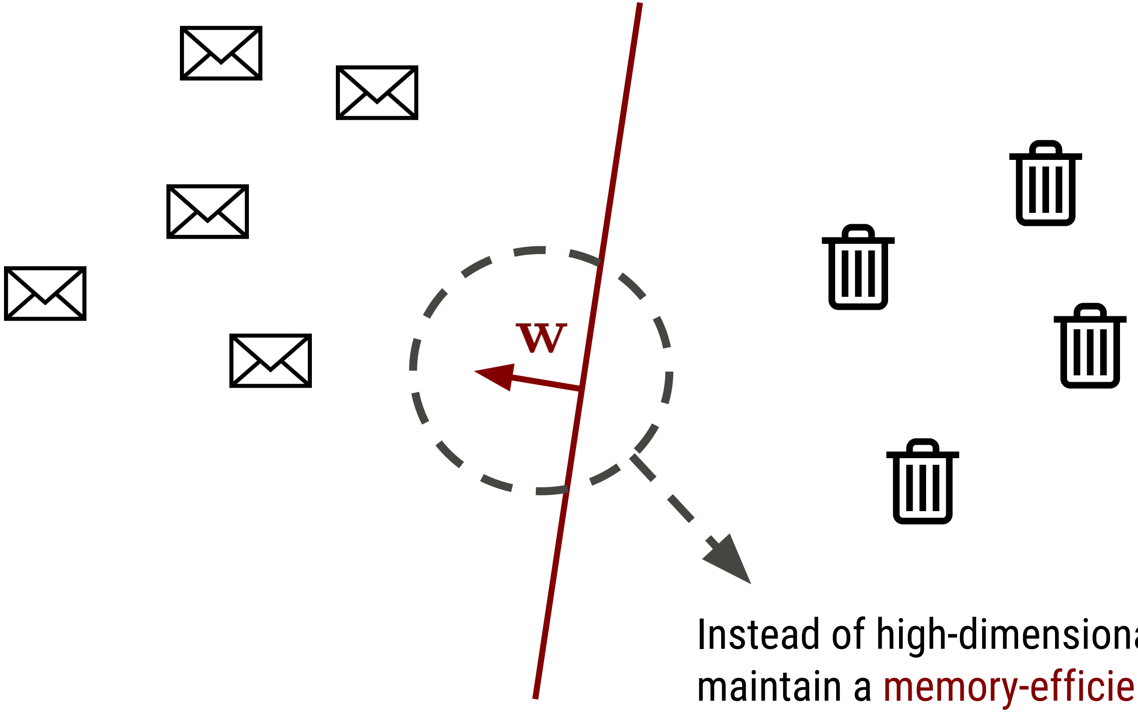
- Count-Sketch [Charikar, Chen & Farach-Colton '02]
- Count-Min Sketch [Cormode & Muthukrishnan '05]

Can we adapt existing sketching algorithms for use in *memory-limited streaming classification*?

**Yes — our contribution.** Weight-Median Sketch: a new sketch for linear classifiers

Main idea: most frequent items $\longrightarrow$ highest-magnitude weights

# This work: Sketched linear classifiers with online updates




Instead of high-dimensional classifier, maintain a memory-efficient sketch

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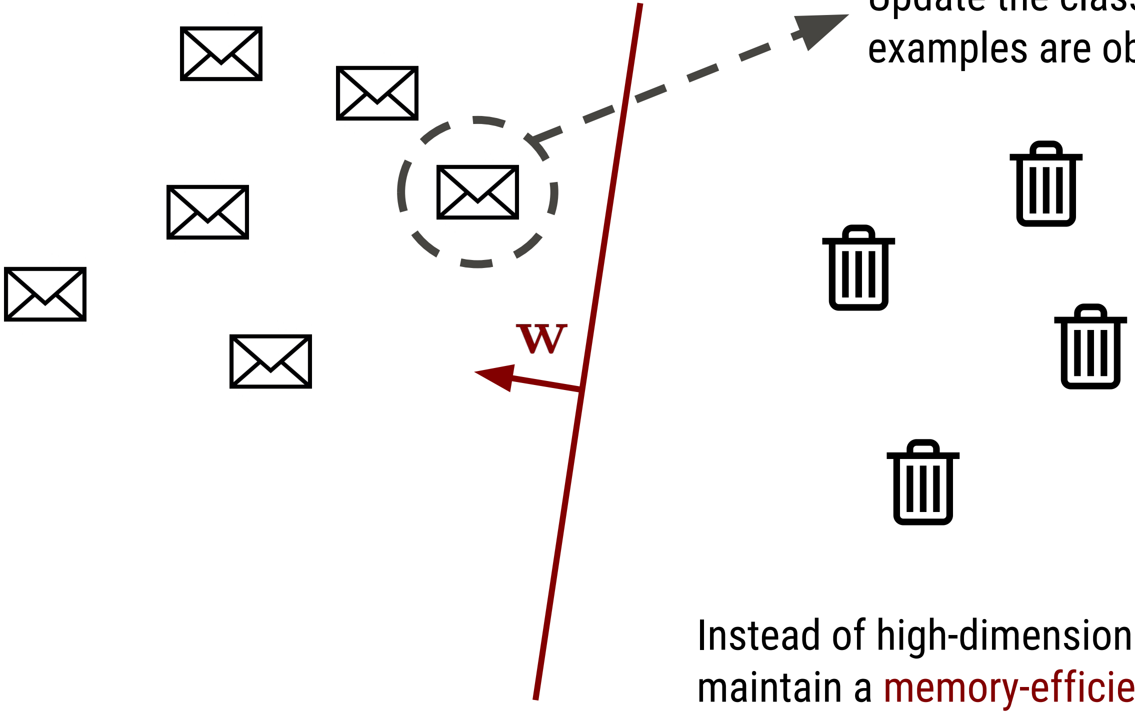


Update the classifier as new examples are observed

Instead of high-dimensional classifier, maintain a memory-efficient sketch

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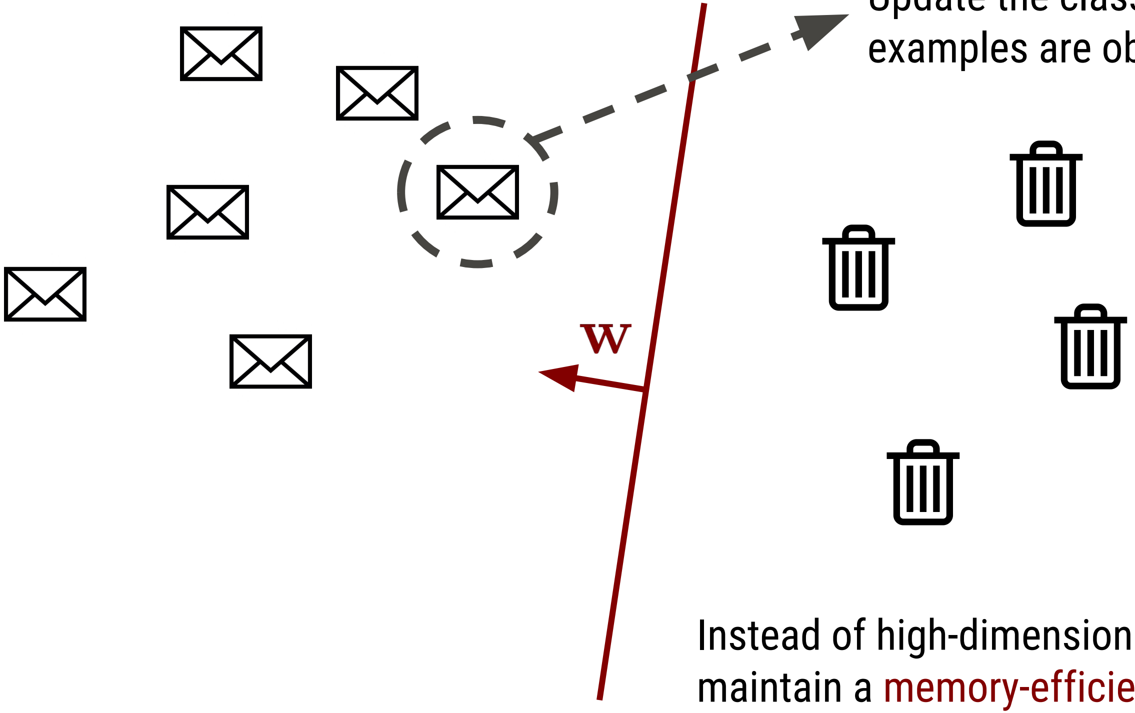


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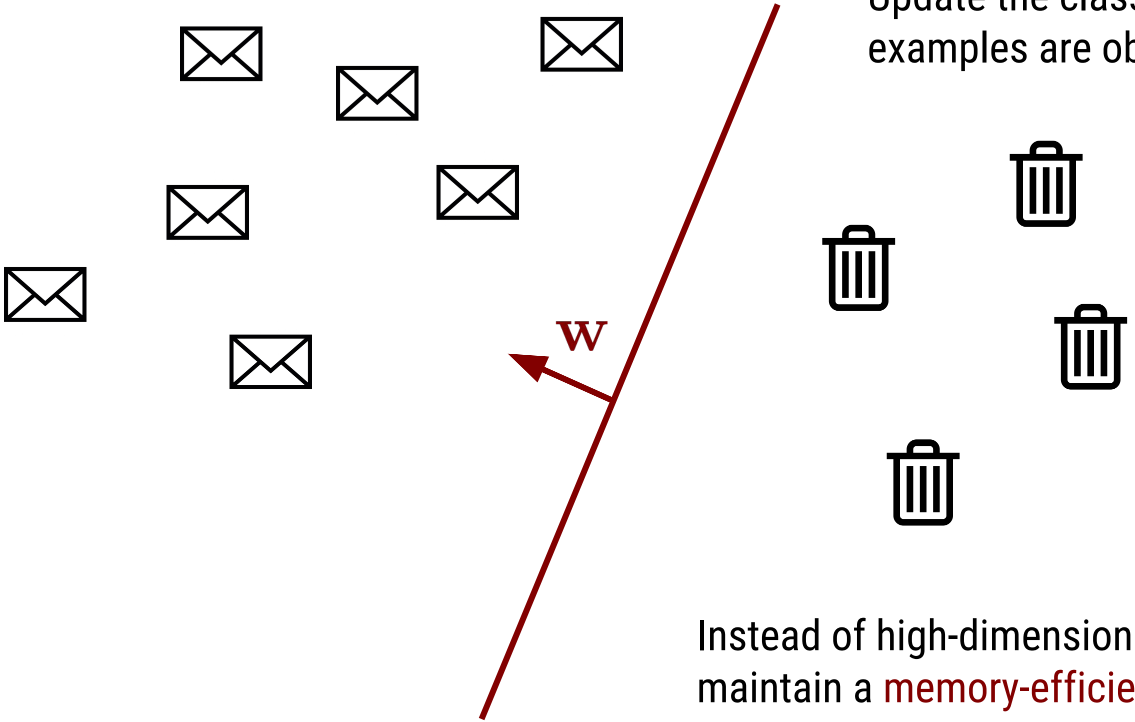


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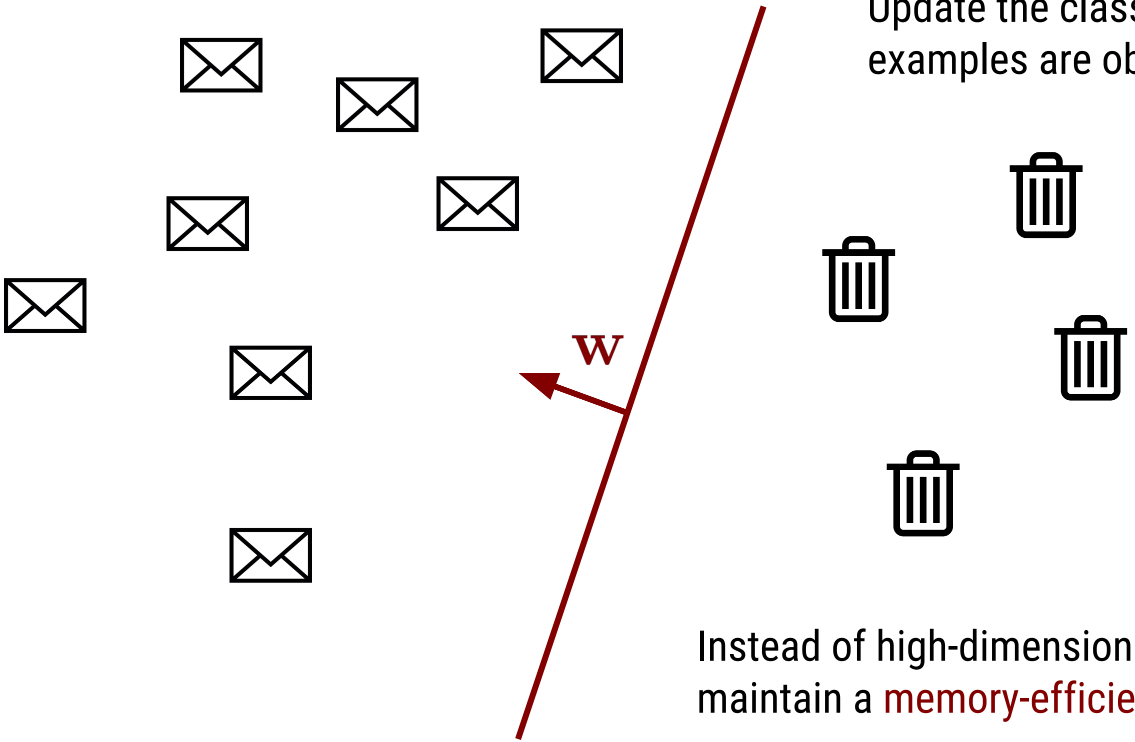


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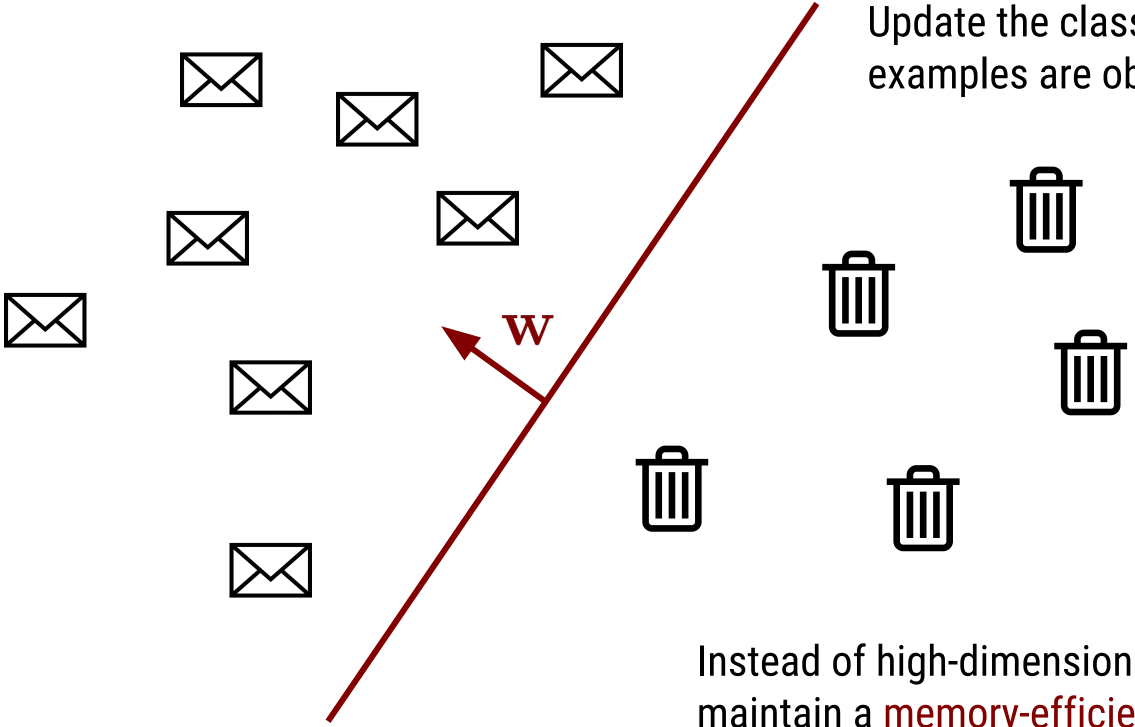


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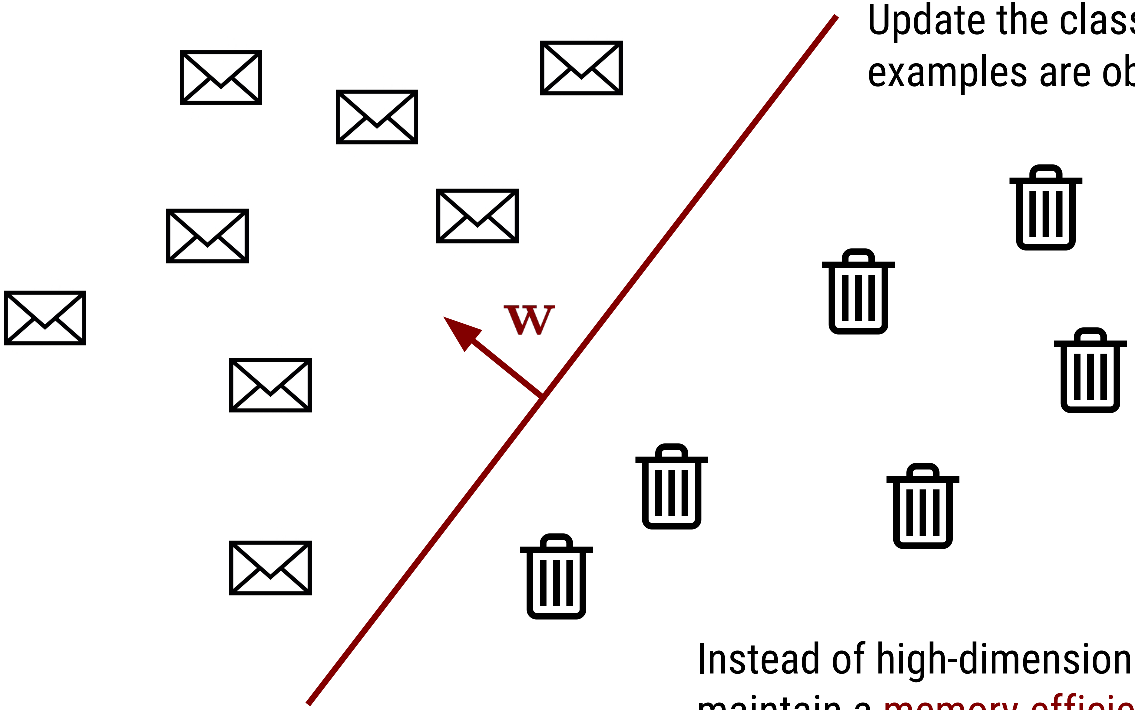


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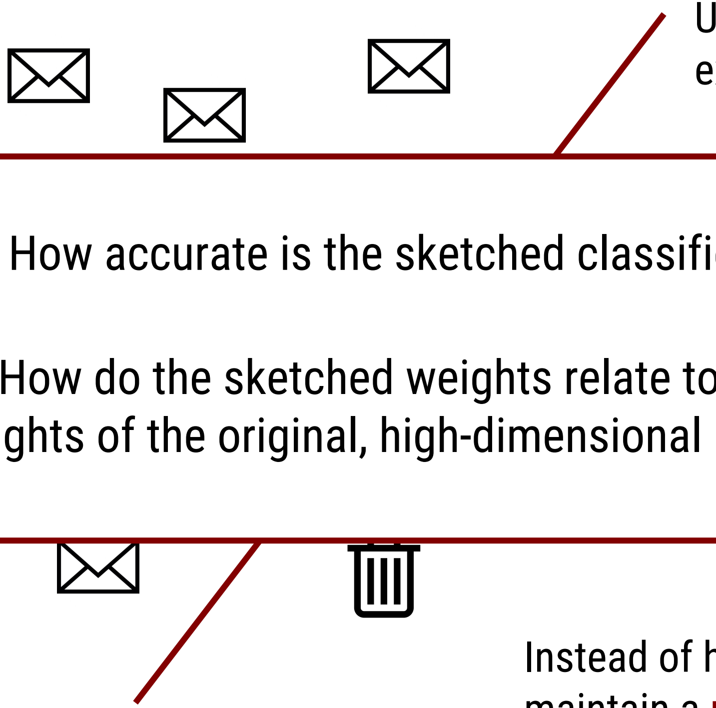


Update the classifier as new examples are observed

How accurate is the sketched classifier?

How do the sketched weights relate to the weights of the original, high-dimensional model?

Instead of high-dimensional classifier, maintain a memory-efficient sketch

# Related Work

## Streaming Algorithms

| | |
|---|---|
| Finding frequent items in data streams | [Charikar et al. ‘02, Cormode & Muthukrishnan ‘05, etc.] |
| Identifying differences between streams | [Schweller et al. ‘04, Cormode & Muthukrishnan ‘05, etc.] |

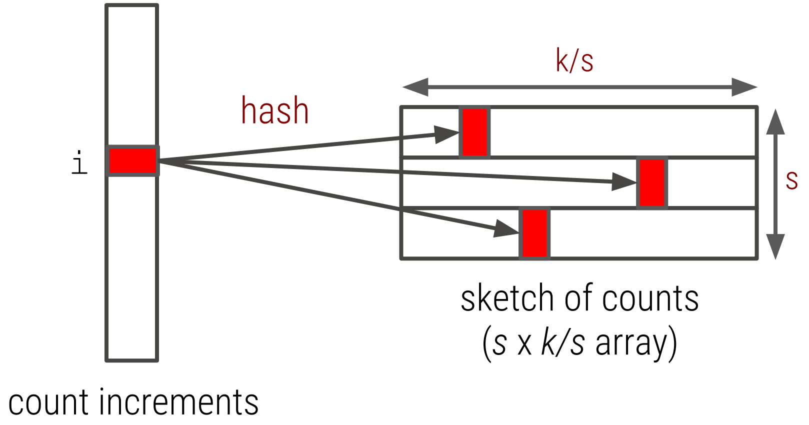
## Machine Learning

| | |
|---|---|
| Resource-constrained learning | [Konecny et al. ‘15, Gupta et al. ‘17, Kumar et al. ‘17] |
| Sparsity-inducing regularization | [Tibshirani ‘96 & many others] |
| Learning compressed classifiers (e.g., feature hashing) | [Shi et al. ‘09, Weinberger et al. ‘09, Calderbank et al. ‘09] |

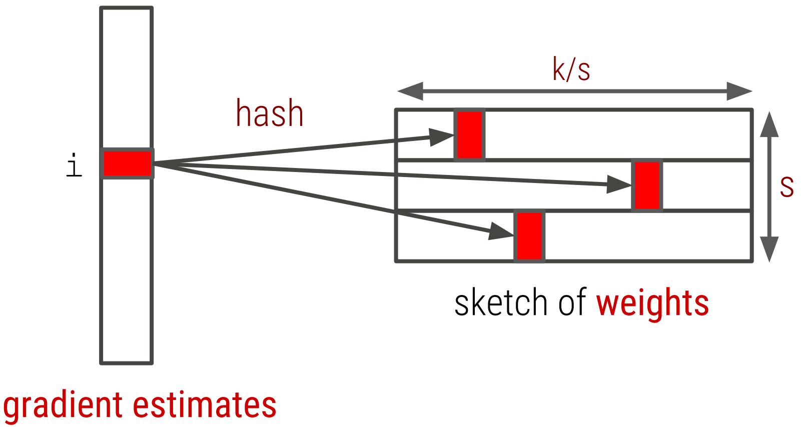
👉 1. algorithm
2. evaluation
3. applications

# The WM-Sketch: an extension of the Count-Sketch

## Count-Sketch update




## WM-Sketch update




Count-Sketch: maintain a low-dimensional sketch of **counts**

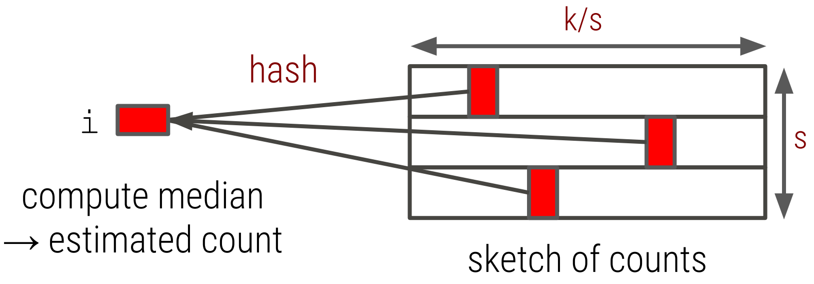
Update: hash each index $i$ to $s$ buckets, apply additive update

WM-Sketch: maintain a low-dimensional sketch of **weights**

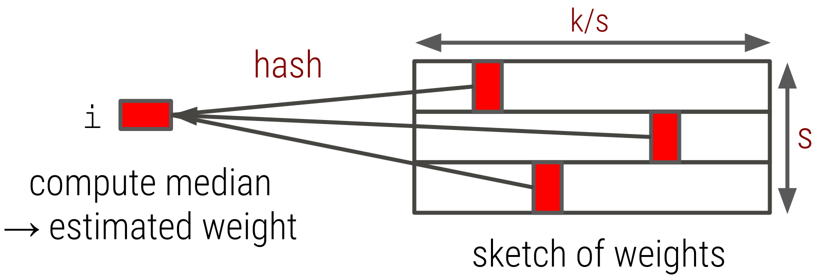
Update: **gradient descent** on sketched weights

# The WM-Sketch: an extension of the Count-Sketch

Count-Sketch query




WM-Sketch query




**Same query procedure**

Count-Sketch → low-error estimates of largest counts

WM-Sketch → low-error estimates of largest-magnitude weights

(note: standard *feature hashing* does **not** support weight recovery)

# WM-Sketch Analysis: Guarantees on weight approximation error

We compare the optimal weights for the **original** data, $w_*$

(i.e., the minimizer of the empirical loss)

to those recovered from the optimal **sketched** weights, $w_{est}$

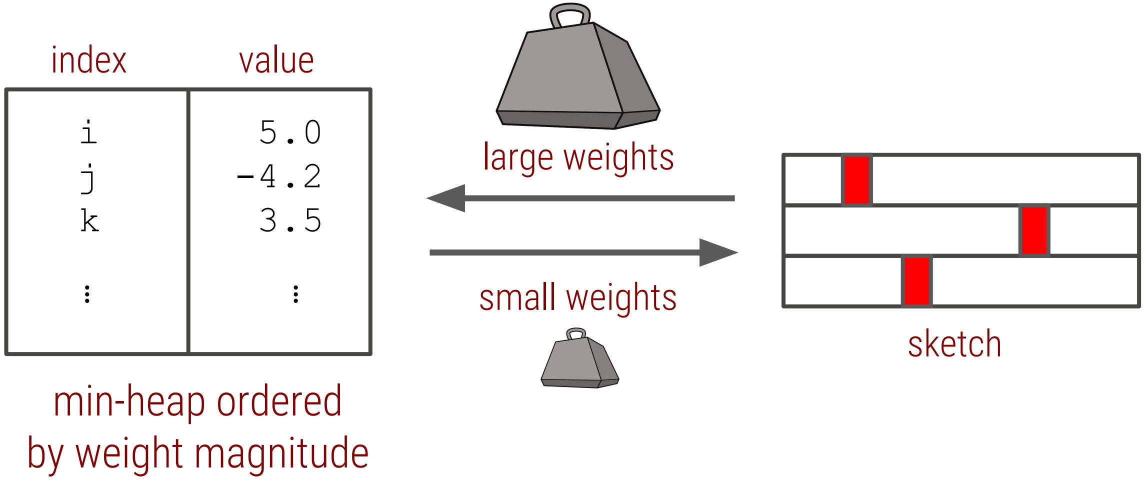
Theorem (informal)

Let *d* be the dimension of the data. With probability $1 - \delta$, the *maximum* entrywise approximation error is $\leq \epsilon \|\mathbf{w}_*\|_1$ for sketch size $\mathrm{O}(\epsilon^{-4} \log^3(d/\delta))$

only need sketch dimension much smaller than *d*

good approximation of high-magnitude weights

# Important optimization in practice: Store large weights in a heap



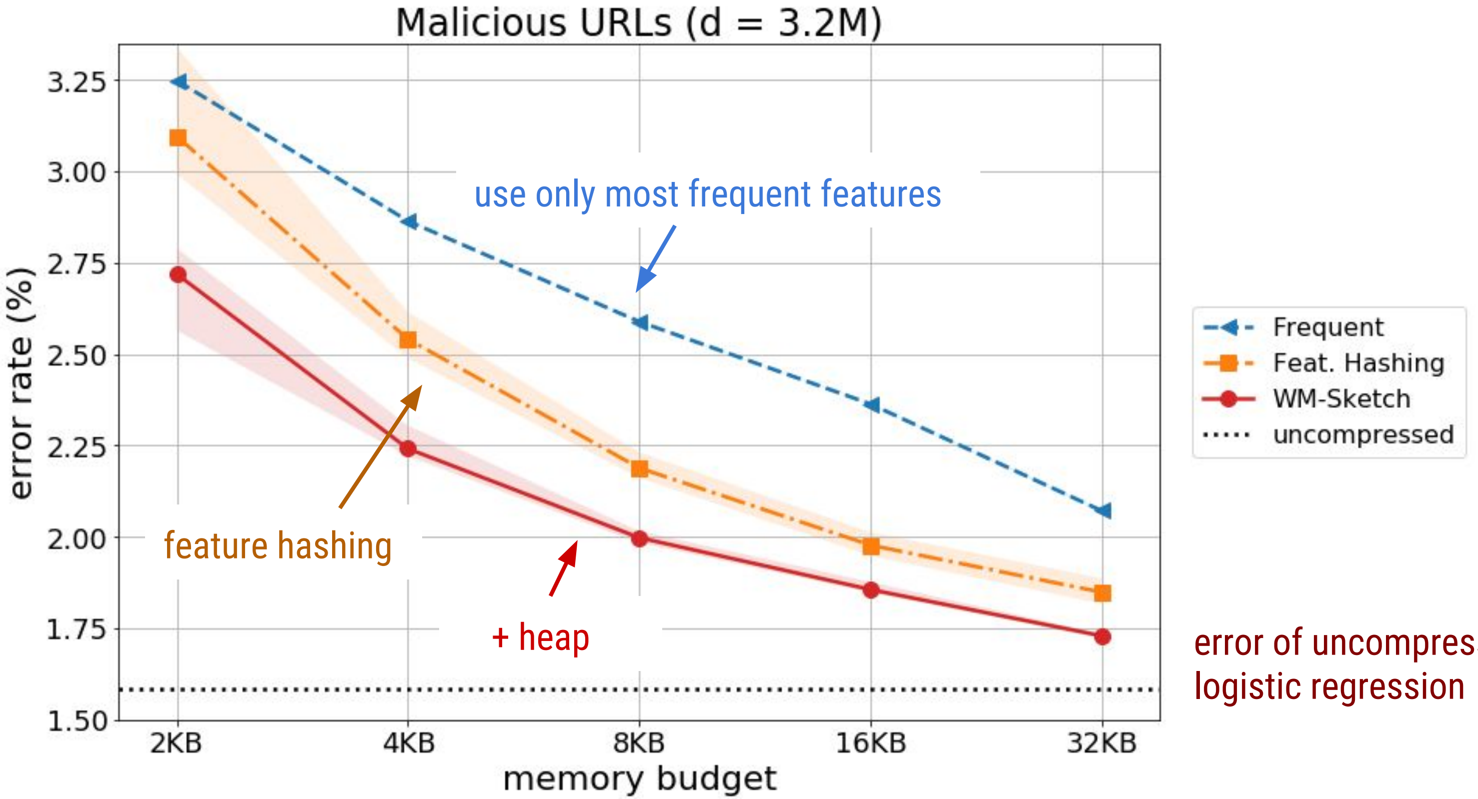

Anytime queries for estimated top-*k* weights

Reduces “bad” collisions with large weights in sketch

Significantly improves classification accuracy
and weight recovery accuracy in practice

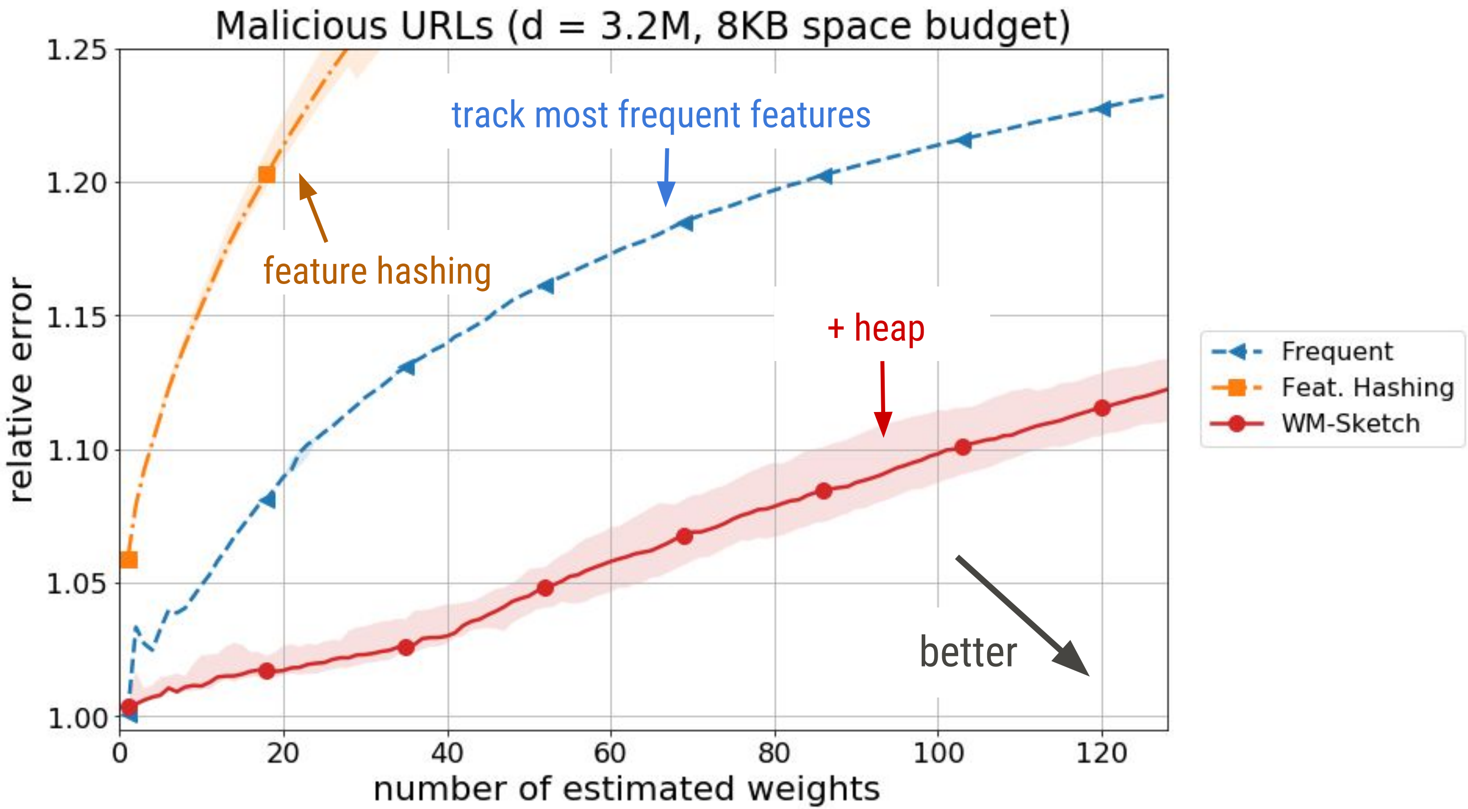
1. algorithm
2. evaluation
   - Classification accuracy
   - Weight recovery accuracy
3. applications

# Classification accuracy: WM-Sketch improves on Feature Hashing




error of uncompressed logistic regression

# Weight recovery: WM-Sketch improves on heavy hitters



1. algorithm
2. evaluation
3. 👉 applications
    - Network monitoring
    - Identifying correlated events

# Network monitoring: what are the largest relative differences?

Network packet

```
Version:    IPv4
Src:   136.0.1.1
Dest:  129.0.1.1
...
```

→

Features

```
 Src[:1] = 136
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 Src[:2] = 136.0
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...
```

→

**logistic regression with WM-Sketch**

Flow A

Flow B

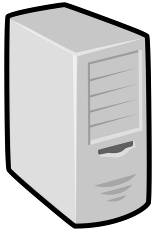
Largest weights → features (e.g., IP prefixes) with largest *relative* differences between flows

Previous work: "relative deltoids" in data streams [CM'05]

Outperforms Count-Min baselines
(even when baselines are given 8x memory budget)

network switch

# Explaining outliers: which features indicate anomalies?



| IP | City | Latency | |
|---|---|---|---|
| 136.0.1.1 | San Francisco | 10ms | label = −1 |
| 161.0.1.1 | New York | 12ms | |
| 129.0.1.1 | Houston | 500ms | label = +1 (e.g. >99th percentile) |
| … | … | … | |

**logistic regression with WM-Sketch**

| Feature | Weight |
|---|---|
| City=Houston | +4.2 |
| City=Austin | +2.0 |
| IP=129.x.x.x | +1.5 |
| … | … |

**Return features most indicative of being an outlier**
(weights can be interpreted as *log-odds ratio*)

**Streaming outlier explanation** (e.g., MacroBase [Bailis et al. '17])

**Outperforms heavy hitter-based methods for identifying "high-risk" features**

# Identifying correlations: which events tend to co-occur?

Text stream →

| Token 1 | Token 2 | Label | |
|---|---|---|---|
| United | States | +1 | real co-occurring events |
| computer | science | +1 | |
| computer | the | −1 | synthetic event pair |
| … | … | | … |

↓

**logistic regression with WM-Sketch**

↓

| Pair | Weight |
|---|---|
| (United, States) | +4.5 |
| (Barack, Obama) | +4.0 |
| (computer, science) | +2.0 |
| … | … |

Features = event pairs

Largest weights → events that are strongly correlated

Exact counter-based approach: 188MB memory
Approximation with WM-Sketch: 1.4MB memory

Approximation ⇒ **>100x less memory usage**

# Takeaways

## Weight-Median Sketch:

- Count-Sketch for linear classification
- Improved feature hashing
- Lightweight, memory-efficient classifiers everywhere


## Stream processing:

- Many tasks can be formulated as classification problems
- Still lots of room for exploration

**Paper:** tinyurl.com/**wmsketch**
**Code:** github.com/stanford-futuredata/**wmsketch**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